WELCOME
Many pay items are computed on the basis of area measurements, items such as base, surfacing, sidewalks, ditch pavement, slope pavement, and Performance turf. This chapter will describe methods for performing these calculations and provide example problems to illustrate documentation for final estimates. The Construction Math training course presents basic information on area measurements. Chapter 8 of that course is entirely devoted to calculating areas. If you have trouble with this chapter of Final Estimates, review Chapter 8 of the math course.

METHODS FOR COMPUTING AREAS
Three methods for computing areas are described in this chapter:

- geometric formulas (including applications of trigonometry);
- latitudes and departures; and
- computer programs.

We will study all three methods, but first let's review a few basic points about units of measurement.

UNITS OF MEASUREMENT
Areas usually are measured in terms of square feet and square yards or acres. Sometimes the field measurements and the computations are in units different from those specified for the pay items, and it is necessary to convert answers from one unit to another. For example, since most field measurements are recorded in linear feet, it is convenient to calculate areas in square feet. But, a conversion must be made when the pay item is in square yards. Just keep in mind these relationships. To convert square feet into acres, divide the number of square feet by 43,560. To convert square inches into square feet, divide the number of square inches by 144. To convert square feet into square yards, divide the number of square feet by 9. All right? Now let's get into the area computation methods, beginning with geometric formulas.

GEOMETRIC FORMULAS
Nearly all areas - even irregular shapes -- can be computed by a mathematical formula or a combination of several formulas. This method is not always the easiest way to determine areas -- but if we understand it, it will help us to understand other methods.

In studying geometric formulas, we will divide our discussion into:

- rectangles, parallelograms, trapezoids;
- triangles (including trigonometric relationships);
- circles (including radians); and
- combinations of shapes.

RECTANGLES, PARALLELOGRAMS, TRAPEZOIDS
These are the most simple area computations and are applicable to many highway features. The basic formula is Area = Length x Height but there are a few special points to remember.

Rectangles are four-sided figures with opposite sides parallel and four 90° angles. A square is a special type of rectangle with all sides of equal length.

Parallelograms also have parallel opposite sides, but the angles are larger or smaller than 90°. For these figures, the height (H) is always measured perpendicular to the base side. Do not use the slope height for computations.

Trapezoids have only two parallel sides. The length used for computation of areas is the average of the lengths of the parallel sides.

Work problem example of a square:

Calculate the area in Square feet for the square shown. Each side = 10 Ft. Answer to the nearest Square Foot. When you are ready to reveal the answer select the Show answer button or select Alt N
Solution: Area for a square equals length times height, which in this example equals 10 feet times 10 feet. This makes the area of this square 100 square feet.
Solution: Area = L x W = 10 Ft. x 10 Ft. = 100 S.F
Work problem example of rectangle:

Calculate to area of the rectangle shown. answer to the nearest Square Yard. When you are ready to reveal the answer select the Show answer button or select Alt N

Solution: The area for a rectangle equals length times height, which in this example equals 20 feet times 10 feet. This makes the area of this rectangle 200 square feet. However, the problem asks for the solution to be given in square yards. Remember that the way to convert square feet is to divide by 9. 200 divided by 9 equals 22.22. The problem also asks that we give our answer in the nearest square yard. We should round our answer, which gives a final answer as 22 square yards.

Work problem example of a Parallelogram:

Calculate the area of the Parallelogram shown. Answer to the nearest Square Foot. When you are ready to reveal the answer select the Show answer button or select Alt N

Solution: The area for a parallelogram equals length times height. In the case of a parallelogram, we need to remember to determine the height based on the perpendicular measurement to the base side. In this example, the area would be 20 feet times 10 feet, making the area 200 square feet.

\[ \text{Area} = L \times H = 20 \text{ Ft.} \times 10 \text{ Ft.} = 200 \text{ Square Feet.} \]

Work problem example of a Trapezoid:

Calculate the area of the Trapezoid shown. Answer to the nearest Square Yard. When you are ready to reveal the answer select the Show answer button or select Alt N

Solution: The area for a trapezoid is the average of the two parallel sides or bases, time the height. The height is determined in the same way for a trapezoid as a parallelogram. In this example, add the two bases 15 ft. and 23 ft. which equals 38. Divide 38 by 2 to get the average of the two bases. Then multiple by the height or 16 ft. This equals 304 square feet. Remember the problem asked for the answer in square yards. Divide 304 by 9 to get 33.78 square yards. Round to the nearest square yard, which gives us a final answer of 34 square yards.

TRIANGLES

Any triangle can be treated as one-half of a rectangle or parallelogram. The area, then, is one-half of the product of the base (B) times the height (H),

Remember, H is measured perpendicular to the base of the triangle not along the slope. Let's find the area of the triangle shown. We will answer to the nearest foot.

Height = 12 inches and Base = 16 inches. Because H is perpendicular to the base B, we can use the equation A equals B times H divided by 2. So, the area would be 16 inches (base) times 12 inches (height), divided by 2. This equals 96 square inches.

Remember that you can find square feet by dividing any value of square inches by 144. 96 divided by 144 equals 0.67 square feet. The problem asks for an answer in the nearest square foot, so we will round our answer to 1 square foot.

To use the Area = Base times Height divided by 2 formula, you must know the height. Here are three examples of triangles that would include the height. Select the continue button or Alt N when you are ready to proceed.

But what if we don't know the height? Well, if we know the lengths of all three sides we can compute the area using this formula: The square root of s times s minus a times s minus b times s minus c.

S equals half of the total sum of all three sides. Simply put, .05 times a+b+c, where a, b, and c are represent the length of the sides of the triangle. Here is example of a triangle where Height is unknown and where all sides are known:

Let's work through a problem. The values for this triangle are a=20 inches, b=28 inches, and c=36 inches. Let's find the area to the nearest square foot. First, we need to calculate the variable “s”. Because we know all three sides, we can add these values and multiply by .5, or take one half of the total. 20 + 28 + 36 = 84. Half of 84 equals 42. Now we have a
value for s. Based on our equation, we now plug in the values for s, a, b, and c. Next we will use order of operations to solve the equation. According to the order of operations, we must first address the items in parantheses, subtracting each side from s. 42-20 = 22. 42-28 = 14. 42-36=6. This leaves us with the square root of 42 times 22 times 14 times 6. Again the order of operations dictates that we must perform our multiplication before we can use the square root. So we will multiply these 4 values to arrive at 77,616. The square root of 77,616 rounded to the second decimal place is 278.60 square inches. The problem asks us to provide our answer in square feet. Using our conversion table, we know to divide square inches by 144 to calculate square feet. The area of the triangle is 1.93 square feet. The problem also asked that we round to the nearest foot. Our final answer then will be 2 square feet.

1) The unit of measurement for item No. 285 – 7 - Optional Base is Square Yards. Which of the following is the area of a 25-foot wide base, constructed between Station 234+20 and Station 295+31, to the nearest square yard?

A. 16,975  
B. 152,775  
C. 16,974.55  
D. 152,774.90

2) The area of the parallelogram shown is 21.55 square feet.

A. True  
B. False

3) The area of the Trapezoid shown is 12.75 square feet.

A. True  
B. False

4) The area of the Triangle shown is 70,300 square feet.

A. True  
B. False

5) The area of the Triangle shown is 492.1 square feet.

A. True  
B. False

6) If an acre has 43,560 square feet, how many feet are in 2 and two-tenths acres?

A. 65,322 Square feet  
B. 95,832 Square Feet  
C. 19, 800 Square Feet  
D. 91,476 Square Feet

Now we will return to the lesson.

TRIGONOMETRIC RELATIONSHIPS

Sometimes it is necessary to use trigonometric relationships to calculate dimensions. We can't cover a complete course in trigonometry but let's review a few of the known relationships that can be helpful.

In the case of a right triangle (where one angle is 90°) we can find the length of any side if we know the length of the other two sides. The known relationship is that the square of the hypotenuse (side opposite the 90° angle) is always equal to the sum of the squares of the other two sides ("adjacent" and "opposite" sides). This is known as the Pythagorean Theorem. For example:

If we know the length of sides a and b, then: \( C = \sqrt{a^2 + b^2} \)
If we know \(a\) and \(c\), then: \(b = \sqrt{c^2 - a^2}\)

If we know \(b\) and \(c\), then: \(a = \sqrt{c^2 - b^2}\)

1) Calculate the length of side \(c\) in the triangle below to the nearest foot.
   - A. 51 Ft.
   - B. 71 Ft.
   - C. 52 Ft.
   - D. 75 ft.

2) Determine which of the following is the length of side \(b\) (to the hundredths of a Foot) and the area (to the nearest Square Foot) of the triangle shown.
   - A. Length \(b = 28.83\) Ft. and Area = 778 Square Feet.
   - B. Length \(b = 26.15\) Ft. and Area = 706 Square Feet.
   - C. Length \(b = 27.46\) Ft. and Area = 741 Square Feet.
   - D. Length \(b = 30.27\) Ft. and Area = 817 Square Feet.
   - E. None of the above.

3) Determine which of the following is the length of side \(c\) (to the nearest foot) and the area (to the nearest square foot) of the triangle shown Note: \(a = b = 17\) Ft.
   - A. \(c = 24\) Ft.; Area = 145 S.F.
   - B. \(c = 21\) Ft.; Area = 355 S.F.
   - C. \(c = 22\) Ft.; Area = 560 S.F.
   - D. \(c = 29\) Ft.; Area = 155 S.F.

**CIRCLES**

When you work with circular areas, remember the following relationships: Pi equals 3.1416 (for our course) or the circumference of a circle divided by its diameter. Therefore, it follows that the Circumference is equal to Pi times the diameter. Because the radius of a circle starts at the center, it is always half of the diameter. Therefore, the diameter is 2 times the radius. There are two methods to find the Area of a circle. One formula states Area is equal to Pi times the circle's radius squared. A slightly more accurate formula, which is preferred, states Area is equal to the radius squared divided by 2 times the radian of angle.

Radians are another way to describe angles, instead of degrees. The radian of 1 degree is equal to Pi divided by 180, or 0.0174533. We will look more into radians a little later.

If the area of a circle is equal to Pi times the radius squared, we can assume that the area of a semi-circle, or half circle, is equal to Pi times the radius squared divided by 2.

To find the area of any other sector of a circle, Multiply Pi by the radius squared by the angle divided by 360 degrees. Again using radians does supply a more accurate answer, so whenever possible use Pi time the radius squared divided by 2 times the radian of angle.

To determine a segment of a circle, we use the area of the circle minus the area of a triangle. Remember, to find the area of a triangle where the height is unknown, we use the formula Area equals the square root of \(s\) which is one half the sum of all the sides of the triangle, times \(s\) minus \(a\), times \(s\) minus \(b\), times \(s\) minus \(c\). We subtract this value from Pi times the radius squared times the angle divided by 360 degrees, Remember also that sides \(a\) and \(b\) of the triangle are equal to the radius.
Ellipses are similar to circles, but are oblong – or egg shaped. A slightly different formula is used to compute the area:

Area of an ellipse is equal to \( \pi \) times the two radii of the ellipse. Unlike a circle, the line from the center of an ellipse to the edge is not always the same depending on which end you use. Notice the diagram lists a capital R, which is the longest radius. The lower case \( r \) represents the smallest radius.

NOTE: If you need clarification about sectors, segments, ellipses, etc. review the Construction Math Training Course.

There may be more than one correct answer to some quiz problems given in this course. In some calculation problems you may not get the same answer, exactly. The differences are probably due to rounding. In practical applications, answers are computed by using the full capacity of a calculator. In this course, however, we will use the following rules of rounding:

- Pi (\( \pi \)) will be rounded to 3.1416
- Converted inches to feet will be rounded to 4 decimal places
- Radians will be rounded to 7 decimal places
- And Trigonometric Functions will be rounded to 4 decimal places

If you follow these rounding procedures, we should get the same answers.

1) Which of the following is the area of the sector in circle “A” shown to the nearest tenth of a square foot?
   A. 653.5 Square Feet
   B. 65.3 Square Feet
   C. 208.0 Square feet
   D. 27.2 Square Feet

2) Which of the following is the area of the shaded segment in circle “B” shown to the tenth of a Square Foot?
   A. 4.4 Square Feet
   B. 1.4 Square Feet
   C. 2.0 Square Feet
   D. 1.9 Square Feet

3) The area of the ellipse shown is 2,890 Square Inches to the nearest Square Inch.
   A. True
   B. False

RADIANS
Usually, a central angle formed between two radii is measured in degrees and we know that there are 360° in a circle. Central angles also can be measured by another unit, called radians. This is more accurate than using degrees.

What is a radian? It’s simply the ratio of the length of an arc of a circle to the length of the radius and it serves as a measure of the central angle between the two radii. This means that if the arc of a sector is equal to the radius, that angle has a measurement of one radian.

How many radians are there in a circle? Because a radian is the angle of a sector of a circle where the arc is equal to the radius, if the circumference equals \( 2\pi r \), or 6.2832r, then there are \( 2\pi \) radians in a circle.
Radians = \frac{\text{Arc}}{\text{Radius}} = \frac{6.2832}{1} = 6.2832 \text{ radians}

How many degrees in a radian?

1 \text{ radian} = \frac{360}{6.2832} = 57.2956° *

How many radians in a degree?

1 \text{ degree} = \frac{6.2832}{360} = 0.0174533 \text{ radians or } \frac{\pi}{180} = 0.0174533

This last value -- 0.0174533 radians in 1 degree -- is a very important number to remember. Memorize it.

1) In constructing a circular curve for a driveway as outlined, the center line radius is 125 Ft., the delta of the curve is 114.5917 degrees, and the roadway width is 30 Ft. With these dimensions, which of the following is the length of the center line to the nearest foot?

A. 2,000 feet
B. 250 feet
C. 289 feet
D. 350 feet

2) In constructing a circular curve for a driveway as outlined, the center line radius is 125 Ft., the delta of the curve is 114.5917 degrees, and the roadway width is 30 Ft. With these dimensions, what is the length in feet of the inside edge of the pavement?

A. 242 Ft.
B. 95 Ft.
C. 155 Ft.
D. 220 Ft.

3) In constructing a circular curve for a driveway as outlined below, the center line radius is 125 Ft. delta of the curve is 114.5917 degrees, and the roadway width is 30 Ft. With these dimensions, the pavement’s surface area is 833.33 Square yards. (To the nearest hundredth of a square yard).

A. True
B. False

COMBINATIONS OF SHAPES

Many irregular areas can be measured readily by breaking the shapes into several component areas, each of which can be computed by a formula. The total area is then found by adding the individual areas -- or sometimes by subtracting one area from another.

For example, the area of a four-sided figure with no sides parallel can be determined by dividing the shape into two triangles and a trapezoid, as shown:

Using the formulas for triangles and trapezoids, the total area is:

Area equals L1 times H1 divided by 2 plus L2 times H1 plus H2 divided by 2 plus L3 times H2 divided by 2.

Okay, let's look at another example. The area of a driveway entrance can be calculated as shown:
The area of the entrance will be the sum of A, B and C, where:

A equals radius squared minus $\pi$ radius squared divided by four. B equals W times radius. And C equals radius squared minus $\pi$ radius squared divided by four.

A simplified approach would be to consider the driveway entrance one rectangle (DEFG) from which the areas of the two quarter-circles (one semi-circle) must be subtracted. Taking the area of a rectangle as Length x Height and subtracting $\pi$ radius squared divided by four.

Many other irregular shapes -- such as concrete slope pavement -- can easily be divided into smaller areas that are regular shapes, which can be calculated individually and then added together.

1) By using the combination-of-shapes method, the total area of the irregular shape shown is 1,758.9 Square Foot (calculate to the tenth of a square foot).

   (Note: Add up all the following: Area of ½ Circle, Area of Triangle, Area of Trapezoid and Area of Triangle).

   A. True
   B. False

2) Which of the following is the area, to the nearest square foot, of the irregular shape shown? (Note: the triangle with the 40° angle, a equals 12 feet and b and c are equal to the radius which is 21 feet.); Hint: Total Area = (Area of Circle – Area of segment) + (Area of Rectangle - Area of ¼ Circle). The formulas are listed below.

A. 1,720 S.F.
B. 935 S.F.
C. 1,865 S.F.
D. 986 S.F.

CURVATURE CORRECTIONS

One factor must be considered when computing areas such as CURVATURE CORRECTION. Corrections for curvature must be made when:

- measurements are determined from a surveyed based line,
- that base line is not the centerline of the area to be measured, and
- the surveyed base line follows a curve.

For example, when computing the area of a two-lane pavement surface, there is no problem as long as the survey line is the center of the highway. The area is found simply by multiplying the stationing length by the surface width. This works on both tangents (straight) and curved sections.

But what happens if the survey line is along the shoulder or the curb line? On tangent sections it makes no difference -- but on a curve the stationing length no longer serves as an accurate basis for computing areas. This is illustrated on the next page.

In the case of the left curve below, the computed area will be less than the actual area if the survey line is used for length measurement. But when the right curve is considered, the computed area will be greater than the actual area.

In other words, when the base line or (survey line) is on the outside of the area with respect to the center of the curve, the computed area will be less than the actual area. When the base line is on the inside, the computed area will be greater than the actual area.

So, what do we do? We introduce a correction factor based on the relationships between the two radii:

Correction factor = $R$ centerline divided by $R$ survey line (to the nearest thousandth)
Suppose that both curves shown had survey line radii of 200 feet, and that the roadway had a 24-foot width. Using the correction factor formula, the left curve correction factor would be: $\frac{188}{200} = 0.94 \ldots$ and the right curve's correction factor would be: $\frac{212}{200} = 1.06$

$200 – 12 = 188$ Ft. (1/2 of 24 Ft. = 12 Ft. to get the radius of the center line for the inside curve, and to get the centerline radius of the outside curve $200+12 = 212$ Ft.)

The computed area between the beginning and end of each curve -- based on survey stationing -- would be multiplied by the appropriate correction factor to determine actual surface area. Suppose that the left curve had survey line radii of 233 feet, the roadway length was 450 feet and the width was 24 foot. What would be the actual surface area of the curve?

$$(450 \text{ ft.} \times 24 \text{ ft.} \times 0.94 = 10,152 \text{ sq. ft.})$$

The same length considered for the right curve would yield:

$$(450 \text{ ft.} \times 24 \text{ ft.} \times 1.06 = 11,448 \text{ sq. ft.})$$

1) The areas of the curved sections of roadway (A) and (B) are 1,189 Square yards and 1,530 Square Yards respectively to the nearest square yards.

   A. True
   B. False

**PERFORMANCE TURF**

Performance Turf is a Plan Quantity Pay Item and is paid in Square Yards

It establishes a stand of grass on slopes, shoulders, or other areas by seeding (includes seeding, seeding & mulching, hydro seeding, bonded fiber matrix, or any combination), or sodding, in accordance with Section 570 of the Specifications.

On projects, this pay item 570- 1 -1 or 570- 1 -2 is coordinated with Sections 104 (Prevention, Control, and Abatement of Erosion & Water Pollution) and 580 (landscaping Installation) of the Specifications.

Plan Quantity Pay Items are not required to be final measured. Only field revisions, and plan errors will be final measured to show what was added or deleted from the plan quantity. There are several approaches we could use. With the stationing and offset distances, we could easily establish coordinates for each corner and compute the area by the method of latitudes and departures. Also, the areas could be broken into several geometric areas, each of which could be computed by a formula and then totaled. Or, we could code some input sheets and let the computer do the work.

If we are concerned with only one area, it would probably be simpler to compute the square yards manually. But if field revisions or the plan errors are significant throughout the project, we certainly should consider using the computer program. This is true for many other items -- simple calculations should be done manually; more complicated or lengthy computations can be done by computer programs.

**COMPUTER PROGRAMS**

Many area computations are relatively simple and can be made easily with a calculator or even manually. However, sometimes manual computations can become difficult because of either the complexity or the large number of calculations. For these situations, the computer programs available from the Department are very helpful in computing and documenting Final Estimate quantities.

Programs currently available for area computations are in the FDOT Quantities program (formally known as the Engineering menu Final Measurement program.)

**LATITUDES AND DEPARTURES**

Latitude and Departure is a method of measurement utilizing offset points that are referenced to a surveyed baseline or centerline of construction to calculate areas. If the area is on a curve, then the baseline follows the curve. This method
averages the widths of each station multiplied by the length between stations to calculate the area. Calculations can be performed manually or by the Department’s FDOT Quantities Program. ALL Latitude and Departure measurements are REQUIRED to be recorded on the Department’s “Final Measurements” Site Source Record (form #700-050-53) or in a bound Field Book, or on the Final Measurement “Miscellaneous” (form 700-050-61). The inspector is required to put their name on this form.

Latitude and Departure measurements are to be taken in the direction of the stationing. That is the first measurement is taken at the lowest station and the following measurements are taken with the stationing, in ascending order. For example:

From 10+00, to 10+50, to 11+00, etc

This does not mean that measurements have to be redone when areas are skipped over during different phases of construction and then returned to at a later date for completion. The measurement would be recorded after the last entry made on the form starting with the lowest station and proceeding forward to the end of that area.

A width measurement must be taken every time the width changes. When widths vary, as with a roadway taper or in curves, more frequent measurements should be taken for accuracy. Be aware of exceptions and station equations. If not noted properly the area will not be calculated accurately.

The following examples will show you how to record measurements on the Latitude and Departure forms.

Performance Turf is a plan quantity item. Measurements are taken only if there is a field revision or a plan error.

In this example, there is a plan error. The designer missed these areas, and construction personnel will need to go out and final measure the necessary areas and document the quantities. An exception begins at Station 20+00 and ends at Station 30+00. The measurements will begin at Station 10+00 and stop at Station 20+00. No measurement is taken until the end of the exception at Station 30+00 where measurements restart and proceed forward. No measurements are taken within the limits of an exception.

This is how Example 1 would be recorded on the Final Measurement form. When you are ready to proceed, select the continue button or press Alt N.

When taking the measurements to be input into the Department's FDOT Quantities Program the calculations in the remarks column are not required. The remarks column should be used to make notations of beginning and ending measurements, intersecting streets, other exceptions or obstructions, and any other pertinent information concerning the measurements.

This example illustrates a Station Equation. The areas 1 and 2 are calculated form Station 10+00 to 20+00. Then areas 2 and 3 are calculated from Station 12+50 to 25+00. It is very important to be sure to calculate these properly otherwise, a substantial error could be made.

Always stop the stationing at the back station and restart with the ahead station.

This is how Example 2 would be recorded on the Final Measurement form. When you are ready to proceed, select the continue button or press Alt N.

1) Which of the following is the area of the Performance Turf to the nearest square yard, using the latitude and departure method?

A) 8,350 SY
   B) 9,345 SY
   C) 7,344 SY
   D) 8,279 SY

2) Using the Latitude and Departure method, the hashed area shows 3.5 inches milling that was left out from the plans. What is the measurement of the hashed area to the nearest square yard?
3) Performance Turf (Sod) is a Plan Quantity pay item, paid for by the Square Yard.

A. True
B. False

4) Field revisions or plan errors on a Performance Turf (Sod) pay item are often measured by offset distances.

A. True
B. False

5) A plan error was noted on a project. The 4 foot sidewalk was not calculated by the designer. Field personnel went out and measured the sidewalk using the Latitude and Departure method. Calculate the missing area from station 10+35 to station 53+21. Rounding your answer to the nearest square yard, which of the following is the missing area?

A. 4,286 SY
B. 476 SY
C. 4,300 SY
D. 480 SY

SUMMARY
Let's review a few of the things you learned about area computations:

- All field measurements should be clearly recorded (odd areas with sketches) in field books or on computer input forms.
- When areas can be determined from simple length - times-width calculations, the preprinted forms for area computation will be sufficient documentation.
- When area computations are more complex, the calculations should be recorded on separate sheets (or on computer input and output sheets) and summarized on the preprinted forms for areas.
- Some irregular areas can be computed by breaking them into several geometric shapes, each of which can be calculated with established geometric formulas.
- The method of latitude and departure or (coordinates of points) can be used to compute the areas of irregular shapes.
- Available computer programs can help reduce the amount of routine manual calculations, improve accuracy and provide reliable documentation of final quantities.

Remember, before final payments can be made, all computations must be checked regardless of which technique is used; the Computation Book must give a complete picture of how the quantities were determined.

1) The area of the driveway shown is 60 Square Yards.

A. True
B. False

2) The area of irregular shapes can be computed by the method of Latitude and departure or (Coordinate of Points).

A. True
B. False

3) All field measurements should be clearly recorded:

A. In the MISCELLANEOUS CONSTRUCTION PROGRAMS MANUAL.
B. With sketches in the Computation Book
C. Odd areas with sketches in the field books or on computer input forms.
D. All the above.
E. None of the above.